SECTION 'B' (Short-Answer Questions) (40)

NOTE: Attempt any Ten part questions in all. All questions carry equal marks. (i.e. 4 marks of each part).

2. i) Solve the quadratic equation: $z^2 - 4z + 5 = 0$ by completing the squares where z is a complex number.

ii) Find the real numbers x, y, z such that matrix A is Hermiation matrix, where $A = \begin{bmatrix} 3 & x+2i & yi \\ 3-2i & 0 & 1+zi \\ yi & i-xi & -1 \end{bmatrix}$

iii) Without expanding determinants, prove that

$$\begin{vmatrix} \alpha & \beta\gamma & \alpha\beta r \\ \beta & \gamma\alpha & \alpha\beta r \\ \gamma & \alpha\beta & \alpha\beta r \end{vmatrix} = \begin{vmatrix} \alpha & \alpha^2 & \alpha^3 \\ \beta & \beta^2 & \beta^3 \\ \gamma & \gamma^2 & \gamma^3 \end{vmatrix}$$

iv) Find λ if the vectors $\hat{\imath} + \hat{\jmath} + 2\hat{k}, \lambda\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} - \hat{k}$ are coplanar.

v) Find n so that $\frac{x^{n+5}+y^{n+5}}{x^{n+4}+y^{n+4}}$ may be $A.M$ between x and y.

vi) Find the sum of the series $3 + 6 + 15 + 42 + 123 + \ldots\ldots$ to n terms.

vii) In how many ways can two English books, three chemistry books and four physics books be arranged on a shelf so that all the books of same subject are together?

viii) Prove the proposition by mathematical induction for every positive integer n : $2 + 4 + 6 + \ldots + 2n = n(n + 1)$

OR

Write in the simplified form the term independent of x in the expansion of $\left(2x - \frac{1}{x^2}\right)^9$.

ix) Find the measure of the largest angle in $\triangle ABC$ with $a = 10cm$, $b = 20cm$ and $c = 26cm$.

x) Solve the following linear programming problem by graphical method when $x \leq 0, y \geq 0$ maximize the objective function $Z(x, y) = 10x + 11y$, subject to constraints $2x + 3y \leq 8; 6x + 3y \leq 10$.

xi) Find the point of intersection of the function $f(x) = x + 4$ and $g(x) = x^2 - 6x + 10$ graphically.

OR

By using graph solve the equation $\cos\theta - \theta = 0$ for the interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

xii) Express $15\sin\theta + 8\cos\theta$ in the form of $r\sin(\theta + \phi)$ where θ and ϕ are in first quadrant.

xiii) Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$.

xiv) Show that: $Tan^{-1}\left(\frac{3}{4}\right) + Tan^{-1}\left(\frac{3}{5}\right) - Tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$.

SECTION "C" (Detailed Answer Questions) (40)

NOTE: Attempt any FIVE question from this section. All questions carry equal marks.

3. Solve the following non-homogenous system of linear equations using Gauss Jordan – Method

$$x - y + 4z = 4$$
$$2x + 2y - z = 2$$
$$3x - 2y + 3z = -3$$

4. A room has 3 lamps from a collection of 1[illegible] light bulbs of which 6 are no good, a person selects 3 at random and puts them in the sockets. What is the probability that he will light?

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5. If $\frac{1}{x} = \frac{2}{5} + \frac{1-3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1-3-5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots$ then show that $4x^2 - 2x - 1 = 0$.

6. If $f : \mathbb{R} \to \mathbb{R}$ is the function defined by $f(x) = 5x + 7$. Find $f^{-1}(x)$ and verify that $f^{-1}[f(x)] = x$.

7. Prove that $\cos20^o \cos40^o \cos60^o \cos80^o = \frac{1}{16}$ by using trigonometric formulae:

8. Find the general solution of the trigonometric equation $\sin4\theta - \sin2\theta - \cos3\theta = 0$ and verify the solution.

9. A, B, C are the points $\vec{a}, \vec{b}$ and $2\vec{a} - \vec{b}$ respectively. D divides $\overline{AC}$ in $2 : 3$ and E divides $\overline{BD}$ in $4 : 1$. Find the position vector of E.